EXAMPLE 9a (part I): nested structure

Markets	Production Sectors X	Production Sectors Y	Production Sectors W	Consumers CONS
PX	120	-20	-100	
PY	-20	120	-100	
PW			200	-200
PL	-40	-60		100
PK	-60	-40		100

The production function in the model is represented as a **nested function**:
- L and K form a Cobb-Douglass aggregate at the bottom level with elasticity of substitution 1:

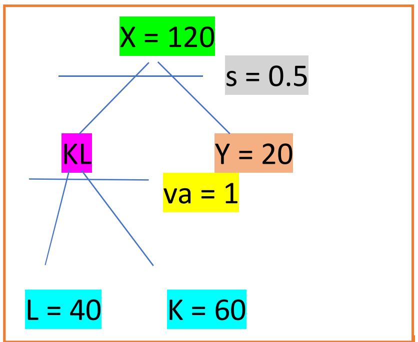
$f(K,L) = A * K^{\alpha} * L^{1-\alpha}$

- At the top level, Y and f(L,K) have an elasticity of substitution equal to 0.5:

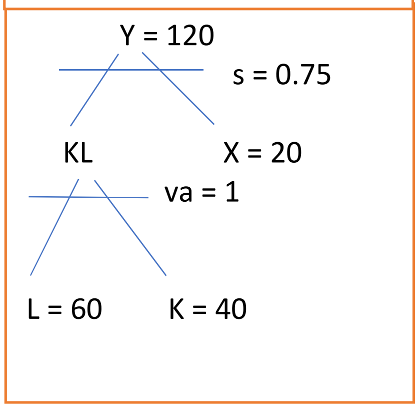
$X = B * [\ \delta Y^{(\sigma-1)/\sigma} + (1-\delta) f(K,L)^{(\sigma-1)/\sigma}\ ]^{\sigma/(\sigma-1)}$

Sector W represents aggregate demand on X and Y by CONS. This means that CONS activity can be described as a "producer" who aggregate single products into composite W and then he consume the composite of products (i.e. not a single product).

```
PROD:X s:0.5    va:1
    O:PX     Q:120
    I:PY     Q:20
    I:PL     Q:40      va:       A:CONS   T:TX
    I:PK     Q:60      va:       A:CONS   T:TX
```




```
$PROD:Y  s:0.75   va:1
         O:PY     Q:120
         I:PX     Q:20
         I:PL     Q:60     va:
         I:PK     Q:40     va:
```




```
$PROD:W  s:1
         O:PW     Q:200
         I:PX     Q:100
         I:PY     Q:100

$DEMAND:CONS
         D:PW     Q:200
         E:PL     Q:100
         E:PK     Q:100
```

```
*1)Benchmark replication
        M1_2S.ITERLIM = 0;
$INCLUDE M1_2S.GEN
        SOLVE M1_2S USING MCP;
```

```
                        LOWER     LEVEL     UPPER    MARGINAL

---- VAR X                .        1.000     +INF       .
---- VAR Y                .        1.000     +INF       .
---- VAR W                .        1.000     +INF       .
---- VAR PX               .        1.000     +INF       .
---- VAR PY               .        1.000     +INF       .
---- VAR PL               .        1.000     +INF       .
---- VAR PK               .        1.000     +INF       .
---- VAR PW               .        1.000     +INF       .
---- VAR CONS             .      200.000     +INF       .
```

Conclusion: Nested structure has no influence on benchmark equilibrium

EXAMPLE 9a (part II): numeraire choice

```
*2)Relax iteration limit
        M1_2S.ITERLIM = 2000;

*Fix the wage rate as numeraire
        PL.FX = 1;

*Counterfactual equilibrium:  100% tax on X sector inputs of K and L
        TX = 1.0;
$INCLUDE M1_2S.GEN
        SOLVE M1_2S USING MCP;
```

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	0.760	+INF	.
---- VAR Y	.	1.173	+INF	.
---- VAR W	.	0.954	+INF	.
---- VAR PX	.	1.719	+INF	.
---- VAR PY	.	1.061	+INF	.
---- VAR PL	1.000	1.000	1.000	1.052E-12
---- VAR PK	.	0.894	+INF	.
---- VAR PW	.	1.350	+INF	.
---- VAR CONS	.	257.541	+INF	.

Technically, after imposing the tax on K & L in sector X ⇒ PK & PL & PX should goes up. However, the results shows that ↓PK. This means that PK relative to PL decreases.

Please note that we implemented two conditions at the same time: (i) new numeraire PL.FX and (ii) imposing tax TX. The first condition has no effect on the benchmark equilibrium, but only on counterfactual equilibrium. The second condition has effect on both, benchmark and counterfactual equilibria, but in order to implement tax in benchmark equilibrium, the calibration process should be modified. It is out of scope for the current exercise.

⇓

let's run this exercise without setting the numeraire (in this case **MPSGE sets a default numeraire - income**[1] of the richest household).

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	0.760	+INF	.
---- VAR Y	.	1.173	+INF	.
---- VAR W	.	0.954	+INF	.
---- VAR PX	.	1.970	+INF	.
---- VAR PY	.	1.216	+INF	.
---- VAR PL	.	1.146	+INF	.
---- VAR PK	.	1.024	+INF	.
---- VAR PW	.	1.548	+INF	.
---- VAR CONS	.	295.118	+INF	

In order to compare both results, we need to divide all price variables by PL in the above table. Relative price PK/PL becomes 0.894 and this is identical result as in the previous run:

$$\frac{1.024}{1.146} = \frac{PK\ when\ default\ numeraire}{PL\ when\ default\ numeraire} = \frac{PK\ when\ PL\ is\ numeraire}{PL\ is\ numeraire} = \frac{0.894}{1}$$

This means that ↑PK after taxation (from 1 to 1.024), but less than PL (from 1 to 1.146), i.e. ↓PK/PL. That's why ↓PK when PL is a numeraire. **How to find that PK increases by 2.4% in the version with PL as a numeraire?** We will show it using market clearance conditions and budget constraint.

[1] It does **not** mean that income becomes fixed, but the income is determined by the current price vector with fixed price relationship.

First, we will show how MPSGE find out price relations when default numeraire is applied. For example, $\frac{P_Y}{P_X}$ we can find using market clearing condition for X:

$$Output + Initial\ Endowment = Intermediate\ Demand + Final\ Demand$$

$$120 \cdot X + 0 = 20 \cdot Y \cdot \left(\frac{P_Y}{P_X}\right)^{0.75} + 100 \cdot W \cdot \frac{P_W}{P_X}$$

$$120 \cdot X = 20 \cdot Y \cdot \left(\frac{P_Y}{P_X}\right)^{0.75} + 100 \cdot W \cdot \frac{P_X^{0.5} \cdot P_Y^{0.5}}{P_X}$$

$$120 \cdot 0.76 = 20 \cdot 1.173 \cdot \left(\frac{P_Y}{P_X}\right)^{0.75} + 100 \cdot 0.954 \cdot \left(\frac{P_Y}{P_X}\right)^{0.5}$$

$$\frac{P_Y}{P_X} \approx 0.61$$

Check: $\frac{P_Y}{P_X} = \frac{1.061}{1.719} \approx 0.61$

Similar analysis we can make to find out $\frac{P_K}{P_L}$ relationship using market clearing condition for K or L (it will be just more complicated to solve it, because market clearing condition for production factors involves also Px and Py).

Second, we will use budget constraint to replicate PK=1.024 under default numeraire:

RA = PK*K + PL*L + TX*(PK*KX + PL*LX) =
= PK(K+ TX*KX) + PL(L + TX*LX)
= PW*W

RA – Households income(RA=257.541),
PK – supplier price of capital after taxation, i.e. net price (PK=0.894),
PL – supplier price of labor after taxation, i.e. net price (PL=1)
TX - tax rate (TX=1)
p_K – purchase price of capital after taxation, i.e. gross price
p_L – purchase price of labor after taxation, i.e. gross price

Relations between prices of sellers and buyers:

$$p_K = \mathrm{PK} \cdot (1 + \mathrm{TX}) = \mathrm{PK} \cdot 2$$

$$p_L = \mathrm{PL} \cdot (1 + \mathrm{TX}) = 1 \cdot 2 = 2$$

In order to produce 120 units of X, we need to use 40 units of labor and 60 units of capital. The results show that only 76% of X capability was used after levying the tax, i.e. 120*0.76=91.2 units of X. However, the amount of inputs is not linearly proportional to output, since the production function is nonlinear (nested CES-CD function):

$$KX = \underbrace{KX_0 \cdot X}_{\text{scale}} \cdot \underbrace{\left[\frac{PX}{PKL \cdot (1 + \mathrm{TX})}\right]^{\sigma}}_{\text{CES function}} \cdot \underbrace{\frac{PKL \cdot (1 + \mathrm{TX})}{PK \cdot (1 + \mathrm{TX})}}_{\text{CD function}}$$

where
X - output
PX – output price
PKL – price of KL composite ($PKL = PL^{0.4} \cdot PK^{0.6}$)
KX - the amount of capital needed to produce 76% of X capability
KX_0 - benchmark amount of capital needed to produce 100% of X capability

$$KX = 60 \cdot X \cdot \left[\frac{PX}{(1+TX)\cdot PL^{0.4}\cdot PK^{0.6}}\right]^{0.5} \cdot \frac{PL^{0.4}\cdot PK^{0.6}}{PK}$$

$$KX = 60 \cdot 0.76 \cdot \left[\frac{1.719}{(1+1)\cdot 1.00^{0.4}\cdot 0.894^{0.6}}\right]^{0.5} \cdot \frac{1.00^{0.4}\cdot 0.894^{0.6}}{0.894} \approx \mathbf{45.7}$$

$$LX = 40 \cdot X \cdot \left[\frac{PX}{(1+TX)\cdot PL^{0.4}\cdot PK^{0.6}}\right]^{0.5} \cdot \frac{PL^{0.4}\cdot PK^{0.6}}{PL}$$

$$LX = 40 \cdot 0.76 \cdot \left[\frac{1.719}{(1+1)\cdot 1.00^{0.4}\cdot 0.894^{0.6}}\right]^{0.5} \cdot \frac{1.00^{0.4}\cdot 0.894^{0.6}}{1.00} \approx \mathbf{27.2}$$

The same we can derive directly from MPSGE using $REPORT command:

```
$REPORT:
          V:KX    I:PK     PROD:X
          V:LX    I:PL     PROD:X
```

The result of the above statement:

```
                                  LOWER      LEVEL      UPPER     MARGINAL

---- VAR KX                 .            45.7497         +INF             .
---- VAR LX                 .            27.2624         +INF             .
```

Now we can insert it into the households income equation:

$$257.541 = PK \cdot 100 \ + \ 1 \cdot 100 \ \ + 1 \cdot (PK \cdot 45.7497 + \ 1 \cdot 27.2624)$$
$$\boldsymbol{p_K = 0.894}$$

We have confirmed results from our first simulation with fixed PL => the above budget definition is proper.

If we do not know prices and income:

$$RA = PK*145.7497 + PL*127.2624$$

$$\frac{RA}{PL*127.2624} = \frac{PK*145.7497}{PL*127.2624} + 1$$

$$\frac{RA}{PL*127.2624} - 1 = \frac{PK}{PL}*1.146$$

Since PK/PL=0.894 and it is the same no matter of numeraire choice, we have

$$\frac{PK}{PL}*1.146 = 0.894*1.146 = 1.024$$

$$PK*1.146 = PL*1.024$$

The above equality requires that PK=1.024 and PL=1.146, i.e. price of K increases by 2.4% => the same result as we obtained when income was a numeraire (by default in MPSGE).

Conclusion: **(i)** We should be careful to add several conditions simultaneously, because conditions may interfere with each other. **(ii)** In our example no interfere take place, but results interpretation is not obvious when one of the prices is fixed as a numeraire. **(iii)** Price change should be interpreted taking into account what is a numeraire in the model. For proper interpretation, the price change should be determined from budget line when no numeraire is set, but price relationships are fixed. **(iv)** Relationships between prices (i.e. relative prices) are the same no matter of numeraire choice. That's why only real variables matter (not nominal variables).

EXAMPLE 9a (part III): welfare quantification

```
*3) Declare a GAMS parameter to hold the solution values
PARAMETER          WELF     Welfare level
                   REPORT   Welfare change;
```

Hicksian welfare is directly observed in MPSGE through the variable that we called 'W' in this model:
`WELF("MOD1")= W.L;`

Welfare is a well-being expressed in monetary units.

It refers to utility gained by possessing goods.

Measured through utility function => Hicksian welfare (*ordinal approach*)
Measured through demand function => Marschallian welfare (*cardinal approach*)

Utility describes usefulness (satisfaction, benefits) of goods, which consumer possess.

Since utility levels cannot be observed directly => we cannot measure it directly (*cardinally*).

Economists assumes that utility can be revealed in willingness to pay different amounts for different goods => we can measure it indirectly (*ordinally*)

The utility each consumer receives from a given amount of income differs. => aggregation is problematic

Consumer **surplus** measures satisfaction in monetary units (while utility - in units of utility).

It is monetary gain obtained by consumers when they purchase a product for a price that is less than they would be willing to pay.

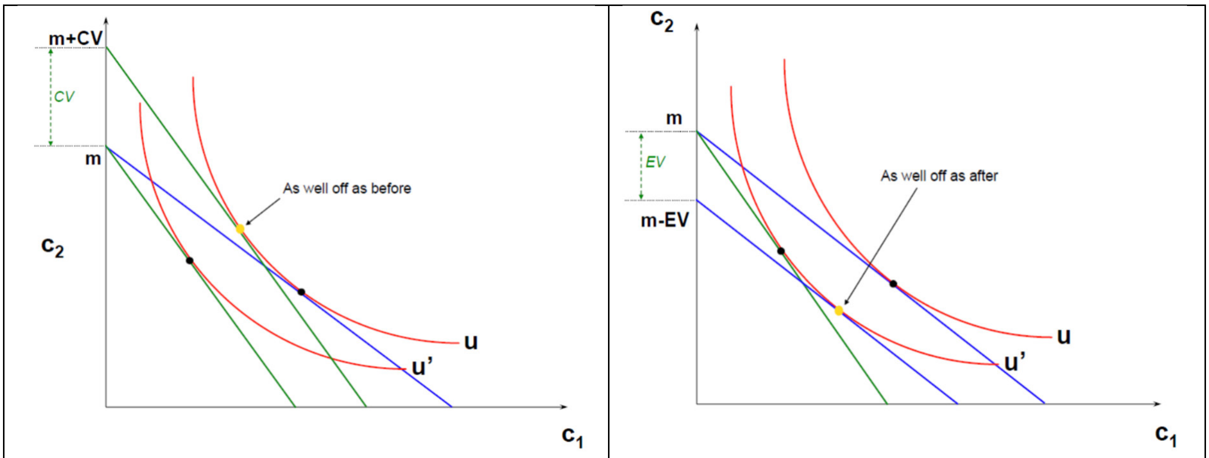
It is measured directly by demand function (*cardinal approach*)

Measures to quantify welfare:

- change in consumer surplus (CS) - *cardinal approach*
- compensating variation (CV) - *ordinal approach*
- equivalent variation (EV) - *ordinal approach*

Compensating variation refers to the amount of additional money a consumer would have to get (due to price change) to make him just <u>as well off as he was before</u>. It is based on Paasche price index.

Equivalent variation refers to the amount of additional money a consumer would have to pay (before the price change) to leave him just <u>as well off as he would be after.</u> It is based on Laspeyres price index.



CV is measured in future (counterfactual) prices | EV is measured in current (benchmark) prices

General relationship:

EV > CS > CV for a price decrease
EV < CS < CV for a price increase

Special case (quasi-linear preferences => no income effect => demand curves are identical):

EV = CS = CV

It's better to measure welfare change through EV, because it does not depend on numeraire choice (since it refers to benchmark prices). CV is numeraire sensitive (since it refers to counterfactual prices).

In MPSGE we can define EV in the following way:

```
REPORT("EX_BURDEN", "MOD1") = 100 * (WELF("MOD1") - 1);
DISPLAY "Compare Excess Burden of Taxation", REPORT;
```

```
----     444 PARAMETER REPORT

                  MOD1
EX_BURDEN       -4.647
```

Conclusion: Since CONS collects taxes (no government), the new tax rate create more benefits for CONS (income goes up) than a harm (price goes up). However, the excess burden of taxation is negative, meaning that households are facing a worse off situation after exposing the tax (income effect < price effect).

Exercise 9a_A:

```
(a). Revise the X sector production to nest Y with K at the bottom(Cobb-Douglas)
level, and then let these inputs trade off with L at the top (CES) nest.
```

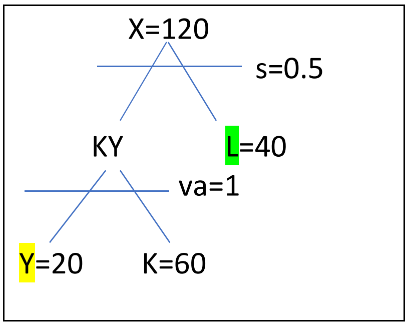
The new nested production function becomes:

- Y and K form a Cobb-Douglass aggregate at the bottom level

$F(Y,K) = A * K^{\alpha} * Y^{1-\alpha}$

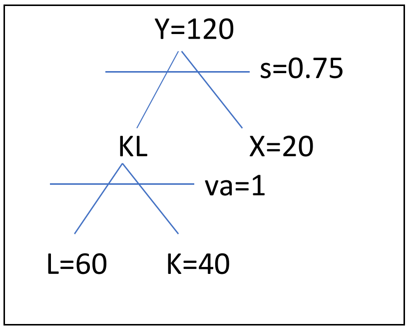
- At the top level, X and f(Y,K) have an elasticity of substitution equal to 0.5:

$X = B * [\ \delta L^{(\sigma-1)/\sigma} + (1-\delta) f(Y,K)^{(\sigma-1)/\sigma}\ ]^{\sigma/(\sigma-1)}$




```
$PROD:X s:0.5   va:1
        O:PX    Q:120
        I:PY    Q:20     va:
        I:PL    Q:40            A:CONS   T:TX
        I:PK    Q:60     va:     A:CONS   T:TX


$PROD:Y s:0.75  va:1
        O:PY    Q:120
        I:PX    Q:20
        I:PL    Q:60     va:
        I:PK    Q:40     va:
```




```
*1)Benchmark replication
        TX = 0;

                        LOWER     LEVEL     UPPER
MARGINAL
---- VAR X                .       1.000     +INF        .
---- VAR Y                .       1.000     +INF        .
---- VAR W                .       1.000     +INF        .
---- VAR PX               .       1.000     +INF        .
---- VAR PY               .       1.000     +INF        .
---- VAR PL               .       1.000     +INF        .
---- VAR PK               .       1.000     +INF        .
---- VAR PW               .       1.000     +INF        .
---- VAR CONS             .     200.000     +INF        .

*2)Counterfactual equilibrium
        TX = 1.0;

                        LOWER     LEVEL     UPPER    MARGINAL
---- VAR X                .       0.766     +INF        .
---- VAR Y                .       1.195     +INF        .
---- VAR W                .       0.950     +INF        .
---- VAR PX               .       1.651     +INF        .
---- VAR PY               .       1.032     +INF        .
---- VAR PL             1.000     1.000     1.000 2.175E-10
---- VAR PK               .       0.841     +INF        .
---- VAR PW               .       1.306     +INF        .
---- VAR CONS             .     248.001     +INF        .


*Extract solution values into this parameter
WELF("MOD2")= W.L;

REPORT("EX_BURDEN", "MOD1") = 100 * (WELF("MOD1") - 1);
REPORT("EX_BURDEN", "MOD2") = 100 * (WELF("MOD2") - 1);

          ----     444 PARAMETER REPORT

                  MOD1               MOD2
EX_BURDEN        -4.647             -5.024
```

Hicks elasticity of substitution measures the percentage change of two inputs of production factors with respect to percentage change in their prices. The new nested structure shows worse results of welfare, because X production is capital intensive and it requires relatively more L than Y.

if $\sigma(K, L) = 1$ => when P_L/P_K increases by 1%, then K/L ratio raises by 1%
$\sigma(KL, Y) = 0.5$ => when P_{KL}/P_Y increases by 1%, then Y/KL ratio raises by 0.5%

if $\sigma(K, Y) = 1$ => when P_Y/P_K increases by 1%, then K/Y ratio raises by 1%.
$\sigma(KY, L) = 0.5$ => when P_L/P_{KY} increases by 1%, then KY/L ratio raises by 0.5%.

Initially producer X could substitute L & K (over 80% of inputs) with the rate 1, i.e. its unit profit condition looks as follows:

$$\left\{\frac{1}{6} \cdot P_Y^{1-\sigma} + \frac{5}{6} \cdot [PKL \cdot (1 + T_X)]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}} = PX$$

$$\left\{\frac{1}{6} \cdot P_Y^{1-\sigma} + \frac{5}{6} \cdot [(P_L^{0.4} \cdot P_K^{0.6}) \cdot (1 + T_X)]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}} = PX$$

where	1/6 = 20/120	share of Y in production of X
	5/6 =(40+60)/120	share of KL composite in production of X
	0.4 = 40/(40+60)	share of L in KL composite
	0.6 = 60/(40+60)	share of K in KL composite
	0.5	elasticity of substitution between Y and KL composite

$$\left\{\frac{1}{6} \cdot (1.061)^{1-0.5} + \frac{5}{6} \cdot [(1.00^{0.4} \cdot 0.894^{0.6}) \cdot (1 + 1)]^{1-0.5}\right\}^{\frac{1}{1-0.5}} = \boldsymbol{P_X = 1.719}$$

Now, elasticity of substitution between L & f(K,Y) is equal to 0.5:

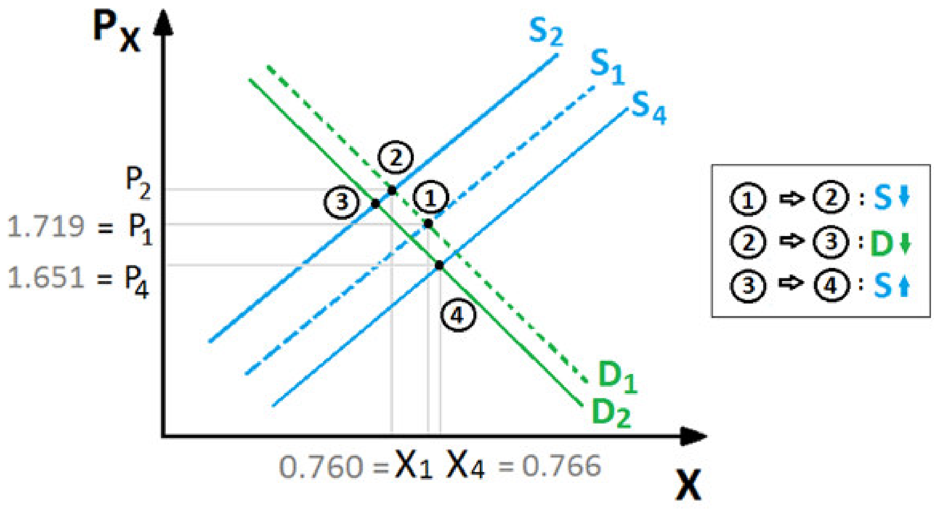
$$\left\{\frac{1}{3} \cdot \left(P_L \cdot (1 + T_X)\right)^{1-\sigma} + \frac{2}{3} \cdot \left[\left(P_Y^{0.25} \cdot \left(P_K \cdot (1 + T_X)\right)^{0.75}\right)\right]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}} = PX$$

where	1/3 = 40/120	share of L in production of X
	2/3 =(20+60)/120	share of K & Y in production of X
	0.25 = 20/(20+60)	share of Y in KY composite
	0.75 = 60/(20+60)	share of K in KY composite
	0.5	elasticity of substitution between L and KY composite

$$\left\{\frac{1}{3} \cdot \left(1.00 \cdot (1 + 1)\right)^{1-0.5} + \frac{2}{3} \cdot \left[\left(1.032^{0.25} \cdot \left(0.841 \cdot (1 + 1)\right)^{0.75}\right)\right]^{1-0.5}\right\}^{\frac{1}{1-0.5}} = \boldsymbol{P_X = 1.651}$$

Thus with the same level of X in both cases, TR=$PX \cdot X$ will be lower in the second case, i.e. worse situation for the producer. When price goes down, consumers increase their demand, thus production of X goes up. However, total revenue becomes anywhere lower than before, i.e. ΔPX>ΔX

This process is divided into 3 parts:
1) supply curve shifts to the left due to substitutability decrease between inputs
2) demand curve shits to the left due to substitution of relatively more expensive X with Y
3) supply curve shifts to the right due to available production capacity (unused K and L by Y)



Now elasticity of substitution between L and f(K,Y) is equal to 0.5 => producer cannot so easily substitute L with K as before, but still $\frac{PK}{PL} < 1$ => producer has to use relatively more L (which is more expensive) and less K to produce X then before => demand for KX↓ => PK↓

$$\mathrm{KX} = 60 \cdot X \cdot \left[\frac{\mathrm{PX}}{PY^{0.25} \cdot \{PK \cdot (1+TX)\}^{0.75}}\right]^{0.5} \cdot \frac{PY^{0.25} \cdot \{PK \cdot (1+TX)\}^{0.75}}{PK \cdot (1+TX)} =$$

$$= 60 \cdot X \cdot PX^{0.5} \cdot \frac{[PY^{0.25} \cdot \{PK \cdot (1+TX)\}^{0.75}]^{0.5}}{PK \cdot (1+TX)} \approx 42.8$$

Using $REPORT command we can display this result directly:

```
                           LOWER          LEVEL          UPPER     MARGINAL

 ---- VAR KX                 .            42.820          +INF          .
---- VAR LX                  .            27.834          +INF
```

Consumes income depends on PL=const, PK↓, KX↓, LX↑:

```
RA  =  PK*K  +  PL*L  +  TX*(PK*KX  +  PL*LX)  =
    =  PK(K+ TX*KX)  +  PL(L  +  TX*LX)=
=  0.841(100  +  1* 42.820)  +  1(100  +  1*27.834)=  248
```

thus RA↓ depends on PK and KX (since PL and LX show different direction). Welfare depends directly on RA => EV↓

Conclusion: Benchmark equilibrium is not sensitive to nested structure, while counterfactual equilibrium is sensitive to it. Direct substitution between K & L gives higher welfare than substitution between K & Y if share of KL > share of KY. Constructing nested function, it is important to group in a single nest the inputs to according to their share and substitutability.

Exercise 9a_B:

```
*Rewrite the original model making an algebraic version
```

The MPSGE code

```
$PROD:X s:0.5    va:1
         O:PX    Q:120
         I:PY    Q:20
         I:PL    Q:40    va:     A:CONS   T:TX
         I:PK    Q:60    va:     A:CONS   T:TX
```

can be rewritten as an algebraic equations:

```
PRF_X..      (20+40+60)*  [1/6*PY**(1-0.5)+5/6*{PL**0.4*PK**0.6*(1+TX)}**  (1-
0.5)]**(1/(1-0.5)) =E= 120 * PX;

where 1/6 = 20/120       is a share of Y in production of X
      5/6 =(40+60)/120   is a share of K & L in production of X
      0.4 = 40/(40+60)is a share of L in KL composite
      0.6 = 60/(40+60)is a share of K in KL composite
```

MPSGE code:

```
$PROD:Y s:0.75   va:1
         O:PY    Q:120
         I:PX    Q:20
         I:PL    Q:60    va:
         I:PK    Q:40    va:
```

Algebraic code:

```
PRF_Y..        120 * [1/6*PX**(1-0.75)+ 5/6*{PL**0.6*PK**0.4}**(1-0.75)]**(1/(1-
       0.75)) =E= 120*PY
```

MPSGE code:

```
$PROD:W s:1
         O:PW    Q:200
         I:PX    Q:100
         I:PY    Q:100
```

Algebraic code:

```
PRE_W..       200 * PX**0.5 * PY**0.5 =E= 200 * PW
```

MPSGE code:

```
$DEMAND:CONS
         D:PW    Q:200
         E:PL    Q:100
         E:PK    Q:100
```

Algebraic code:

```
I_CONS..         CONS =E=100*PL + 100*PK +
                     TX*100*X*PL**0.4*PK**0.6*[PX/((1+TX)*PL**0.4*PK**0.6)]**0.5;
```

```
EQUATIONS
        PRF_X   Zero profit for sector X
        PRF_Y   Zero profit for sector Y
        PRF_W   Zero profit for sector w (Hicksian welfare index)

        MKT_X   Supply-demand balance for commodity X
        MKT_Y   Supply-demand balance for commodity Y
        MKT_L   Supply-demand balance for primary factor L
        MKT_K   Supply-demand balance for primary factor K
        MKT_W   Supply-demand balance for aggregate demand

        I_CONS  Income definition for CONS;

* Zero profit conditions: Cost of Production Gross of Tax = Value of Output

PRF_X..      120* [1/6*PY**(1-0.5)+
                    5/6*{PL**0.4*PK**0.6*(1+TX)}** (1-0.5)]**(1/(1-0.5))
            =E= 120 * PX;

PRF_Y..     120 * [1/6PX**(1-0.75)+
                   5/6*{PL**0.6*PK**0.4}**(1-0.75)]**(1/(1-0.75))
            =E= 120*PY;

PRF_W..      200 * PX**0.5 * PY**0.5 =E= 200 * PW;

* Market clearance conditions: Output + Initial Endowment = Intermediate +
Final Demand

MKT_X..    120 * X =E= 20 * Y * (PY/PX)**0.75
                    + 100 * W * PX**0.5 * PY**0.5 / PX;

MKT_Y..    120 * Y =E= 20 * X * (PX/PY)**0.5
                    + 100 * W * PX**0.5 * PY**0.5 / PY;

MKT_W..    200 * W =E= CONS / PW;

MKT_L..    100 =E= 40 * X * [PX/((1+TX)*PL**0.4*PK**0.6)]**0.5
                                *PL**0.4*PK**0.6 / PL +
                   60 * Y * [PY/(        PL**0.6*PK**0.4)]**0.75
                                *PL**0.6*PK**0.4 / PL;

MKT_K..     100 =E= 60 * X * [PX/((1+TX)*PL**0.4*PK**0.6)]**0.5
                                *PL**0.4*PK**0.6 / PK +
                    40 * Y * [PY/(        PL**0.6*PK**0.4)]**0.75
                                *PL**0.6*PK**0.4 / PK;

* Income balance: the level of expenditure (CONS) = the value of factor
income + tax revenue

I_CONS..         CONS =E= 100*PL + 100*PK +
TX*100*X*PL**0.4*PK**0.6*[PX/((1+TX)*PL**0.4*PK**0.6)]**0.5;

*       We declare the model using the mixed complementarity syntax
*       in which equation identifiers are associated with variables.

MODEL ALGEBRAIC / PRF_X.X, PRF_Y.Y, PRF_W.W, MKT_X.PX, MKT_Y.PY, MKT_W.PW,
                  MKT_L.PL, MKT_K.PK, I_CONS.CONS /;

* Check the benchmark:
        X.L=1; Y.L=1; W.L=1; PX.L=1; PY.L=1; PL.L=1; PK.L=1; PW.L=1; CONS.L=200;
        TX=0;

        SOLVE ALGEBRAIC USING MCP;
```

Note that if no price variable is fixed, a solver may not find a solution when the model is formulated in algebraic form because the Jacobian is singular at the solution (while MPSGE works without any problem because MPSGE uses default normalization – income of the richest consumer, while GAMS has no default normalization).